Let Δ be an appropriate indexing set and for $d \in \Delta$:

M_d a module,

$\mathcal{E}_d \subseteq M_d$,

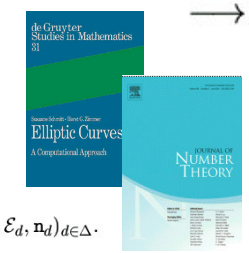
$\mathfrak{n}_d : \mathcal{E}_d \to \bigoplus_{t<d} M_t$ a mapping.

Then we call the module $\mathcal{L} = N/Q$ with

$$N = \bigoplus_{t\in\Delta} M_t,$$

$$Q = \sum_{t\in\Delta} \langle r + \mathfrak{n}_t(r);\ r \in \mathcal{E}_t \rangle$$

the **combination** of the system $\Gamma = (M_d, \mathcal{E}_d, \mathfrak{n}_d)_{d\in\Delta}$.



Theorem 3 *If $\widehat{B_d} \subseteq C^{(n)}$ defines a basis of $\widehat{C^{(d)}}$ for $d|n$ then $B_n = \bigcup_{d|n} \widehat{B_d}$ is a basis of $C^{(n)}$.*

$$\longrightarrow \bigcup_{d\in\mathbf{N}} \widehat{B_d} \text{ defines a basis of } C^{(\infty)} := \bigcup_{d\in\mathbf{N}} C^{(d)}.$$

— Applications of Cyclotomic Units —

Cyclotomic Fields (Algebraic Number Theory):

$[\mathbf{Z}[\epsilon_n]^* : C^{(n)}] < \infty$

$(h_n = 1 \Rightarrow [\mathbf{Z}[\epsilon_n]^* : C^{(n)}] = 1)$

Used in Kummer's approach to FLT:

$$z^p - y^p = \prod_{a=0}^{p-1}(z - \epsilon_p^a y) = x^p$$

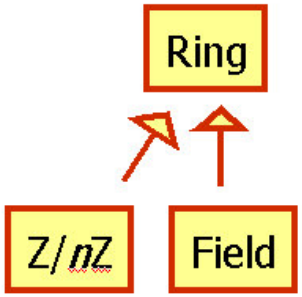
Units in cyclic grouprings (K. Hoechsmann, 1986ff):

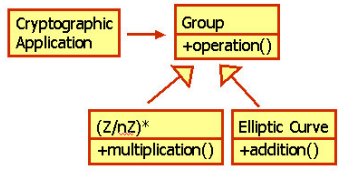
$\mathbf{Z}C_n \cong \mathbf{Z}[x]/x^n - 1 \xrightarrow{x \mapsto \epsilon_n} \mathbf{Z}[\epsilon_n]$

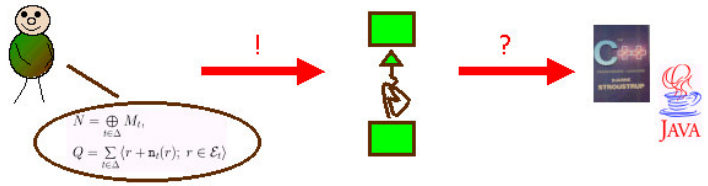
Axiomatic Mathematics

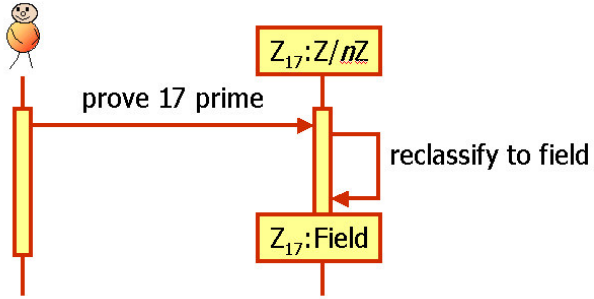
meets

Object Oriented Programming

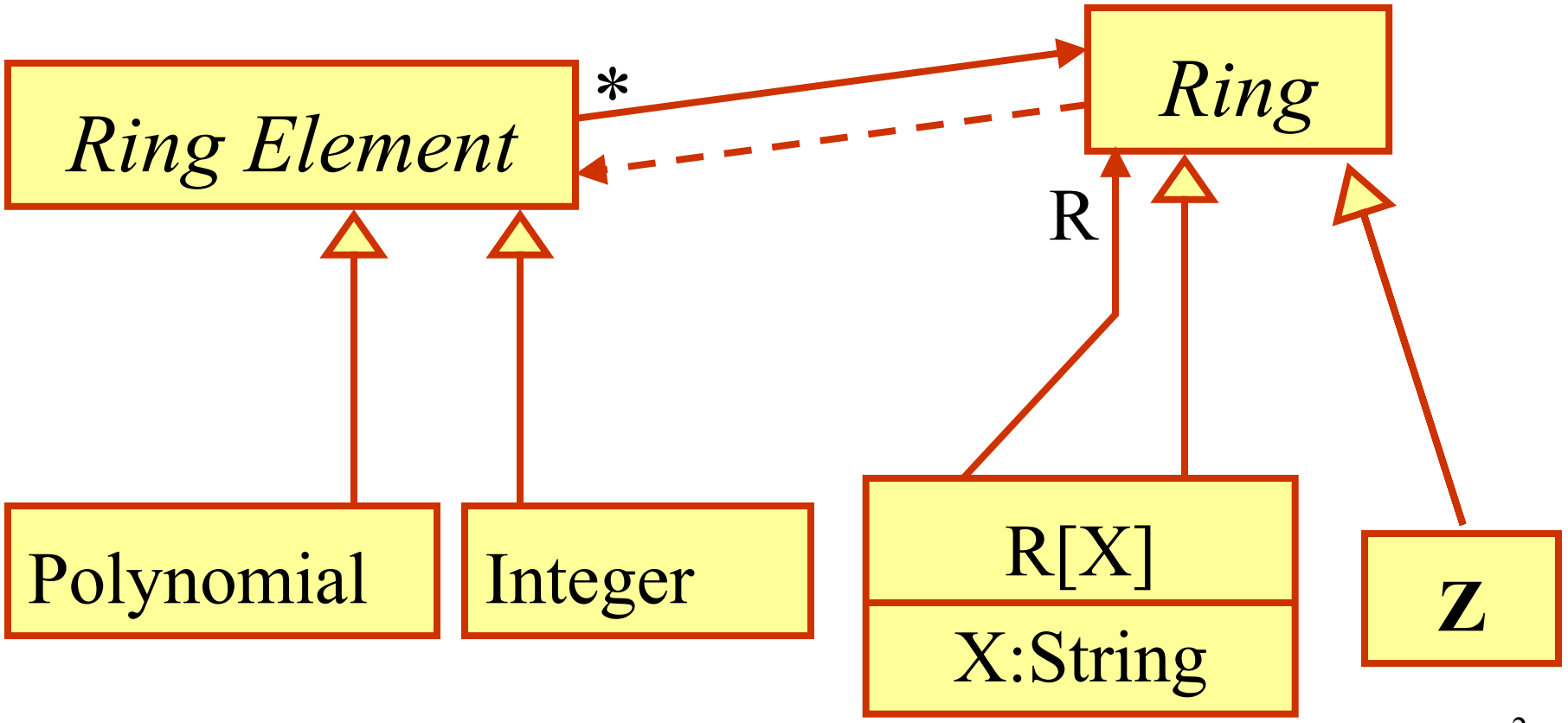








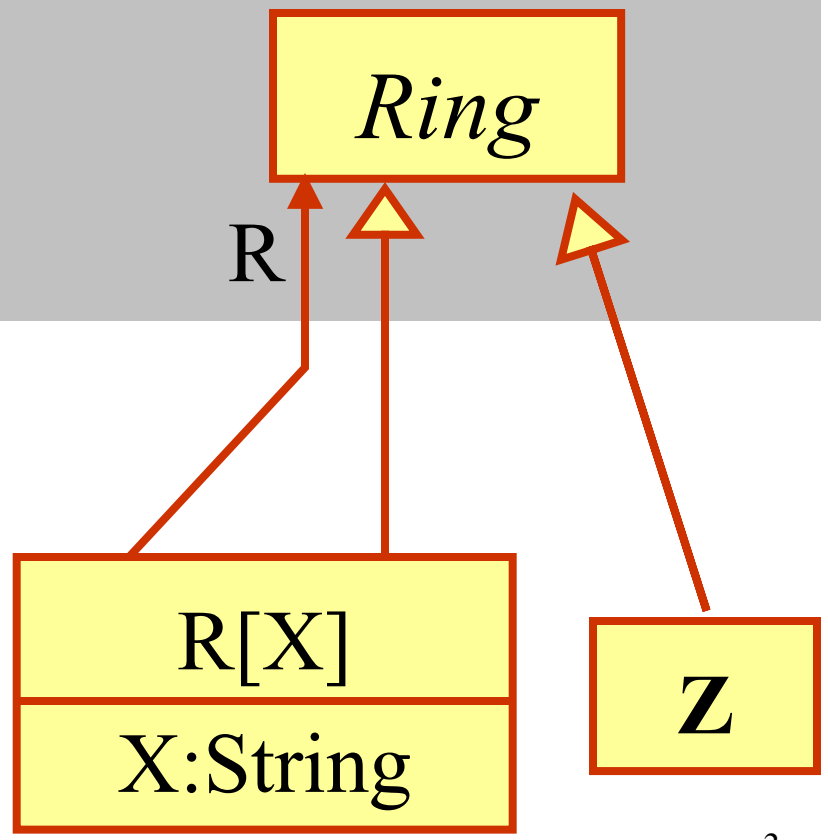
Use the GoF Mediator pattern for Implementing Abstract Mathematics.



Abstract Structures are modelled as abstract classes.

Abstract: An arbitrary (unspecified) ring

Concrete: Ring of Integers, Polynomial Rings, …



Axiomatic definitions are implemented as abstract methods.

Example: *Ring*

- Abstract:
  - addition
  - negation
  - multiplication
  - inversion
  - "zero"
  - "one"
  - check if zero
- Not abstract:
  - subtraction
  - exponentiation
  - embedding of Z and Q
  - Check for equality
  - evaluation of polynomials

The mediator pattern is used in the Java com.perisic.ring package for:

- Rings, Polynomial Rings, Integers, Rational Functions, Algebraic Extensions, Cyclotomic Fields, Universal Rings, etc.

But it can be applied to *any* mathematical structure, such as:

- groups, metric spaces, topological spaces, group rings, etc.

Further Reading

- The Java package com.perisic.ring is available at: http://ring.perisic.com
- M. Conrad, T. French, *Exploring the synergies between the Object-Oriented paradigm and Mathematics: A Java led approach*, to appear in Int. J. Math. Educ. Sci. Technol.
- M. Conrad, T. French, C. Maple, S. Pott, *Mathematical Use Cases lead naturally to non-standard Inheritance Relationships – How to make them accessible in a mainstream language?*, MASPEGHI 2004 (WS 12 of ECOOP)

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